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EFFECTS OF MAGNETIC FIELD AND VARIABLE VISCOSITY ON FORCED NON-DARCY FLOW ABOUT A FLAT PLATE WITH VARIABLE WALL TEMPERATURE IN POROUS MEDIA IN THE PRESENCE OF SUCTION AND BLOWING

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This paper presents a study of the effect of a magnetic field and variable viscosity on steady two-dimensional laminar non-Darcy forced convection flow over a flat plate with variable wall temperature in a porous medium in the presence of blowing (suction). The fluid viscosity is assumed to vary as an inverse linear function of temperature. The derived fundamental equations on the assumption of small magnetic Reynolds number are solved numerically by using the finite difference method. The effects of variable viscosity, magnetic and suction (or injection) parameters on the velocity and temperature profiles as well as on the skin-friction and heat-transfer coefficients were studied. It is shown that the magnetic field increases the wall skin friction while the heat-transfer rate decreases.

Introduction. The motion of a conducting fluid in an electromagnetic field has many physical applications. It is worth mentioning that non-Darcy forced flow boundary layers form a very important group of flows, which are of great importance in many applications, such as biomechanical problems (for example, blood, flow in the pulmonary alveolar sheet) and filtration transpiration cooling.

Previously, idealized models have been developed and used to study the individual as well as coupled effects of flow parameters [1–6]. Elbashbeshy [7] studied the free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of a magnetic field. Most of the analytical studies of this problem are based on the assumption of constant physical properties of the ambient fluid. However, it is known that these properties, in particular, fluid viscosity, can vary with temperature [8]. To accurately predict the flow characteristics and heat-transfer rates, it is necessary to take into account this variation of viscosity.

In the present paper, we consider the effect of blowing (suction) on non-Darcy forced convection flow over a flat plate with variable wall temperature placed in a porous medium, taking into account the variation of the viscosity with temperature in the presence of a magnetic field. The transformed governing non-linear partial differential equations are approximated by non-linear ordinary differential equations replacing the derivatives along the plate by two-point backward finite differences. The results are presented in terms of dimensionless velocity and temperature, the local skin friction coefficient, and local Nusselt number over a wide range of various parameters. The present analysis is an extension of a previous study [9].

Analysis. Let us consider the effect of blowing (suction) on the steady, two-dimensional, laminar, electrically conducting, incompressible, non-Darcy, forced convection flow over a surface embedded in a porous medium under the influence of a transversely applied magnetic field. As the conductivity and, hence, the magnetic Reynolds number are very small, we assume that the induced magnetic field, the external electric field, and the electric field due to polarization of charges are negligible. The fluid properties are assumed to be isotropic and constant, except for the fluid viscosity, which is assumed to be an inverse linear function of temperature (see [10, 11]):

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$$1/\mu = (1/\mu_\infty)[1 + \gamma(T - T_\infty)] \tag{1}$$

or

$$1/\mu = \beta(T - T_r). \tag{2}$$

Here

$$\beta = \gamma/\mu_\infty, \qquad T_r = T_\infty - 1/\gamma.$$

Both β and T_r are constant and their values depend on the reference state and the thermal property of the fluid, i.e., γ. Generally, $\beta > 0$ for liquids and $\beta < 0$ for gases. The free-stream velocity U_∞ and the free-stream temperature T_∞ are uniform.

The governing equations of this problem are written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \tag{3}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \frac{1}{\rho}\,\frac{\partial}{\partial y}\Big(\mu\,\frac{\partial u}{\partial y}\Big) - \frac{\nu}{k}\,(u - U_\infty) - \frac{F}{\sqrt{k}}\,(u^2 - U_\infty^2) - \frac{\sigma B^2}{\rho}\,u; \tag{4}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial y^2}. \tag{5}$$

Here x and y are coordinates along and normal to the plate, respectively, u and v are the velocity components in the x and y directions, respectively, α and ν are the thermal diffusivity and the kinematic viscosity, respectively, k is the permeability of the porous medium, F is an empirical constant in the second-order resistance zone, T is the temperature of the fluid and porous medium, μ is the variable dynamic viscosity, σ is the electrical conductivity, ρ is the density, and B is the magnetic-field strength.

The boundary conditions are defined as follows:

$$\begin{aligned} &y = 0,\ x > 0\colon \quad v = v_w(x) = ax^{-1/2}, \quad u = 0, \quad T = T_w(x) = T_\infty + Ax^\lambda, \\ &y \to \infty\colon \qquad\qquad\qquad\qquad u \to U_\infty, \quad T \to T_\infty. \end{aligned} \tag{6}$$

Here v_w is the surface mass flux and a and A are constants ($A > 0$).

The second and third terms on the right side of Eq. (4) stand for the first-order (Darcy) resistance and second-order (porous inertia) resistance, respectively.

Using the variables

$$\begin{gathered} \eta = (U_\infty x/\nu)^{1/2}(y/x), \quad \psi = (U_\infty \nu x)^{1/2} f(\xi,\eta), \quad \xi = \nu x/(U_\infty k), \\ \theta(\xi,\eta) = \frac{T - T_\infty}{T_w - T_\infty} = \frac{T - T_\infty}{Ax^\lambda}, \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \end{gathered} \tag{7}$$

Eqs. (4) and (5) are reduced to the form

$$f''' - \frac{\theta' f''}{\theta - \theta_r} - \Big(\frac{\theta}{\theta_r} - 1\Big)\Big[\frac{ff''}{2} - Mf' + \xi(1 - f') + \xi \mathrm{Re}_k(1 - f'^2)\Big] = \xi\Big(\frac{\theta}{\theta_r} - 1\Big)\Big(f''\,\frac{\partial f}{\partial \xi} - f'\,\frac{\partial f'}{\partial \xi}\Big); \tag{8}$$

$$\theta'' + \frac{1}{2}\,\mathrm{Pr}\,f\theta' - \lambda \mathrm{Pr}\,f'\theta = \mathrm{Pr}\,\xi\Big(f'\,\frac{\partial \theta}{\partial \xi} - \frac{\partial f}{\partial \xi}\,\theta'\Big). \tag{9}$$

The boundary conditions (6) take the form

$$f(\xi,0) = f_w, \quad f'(\xi,0) = 0, \quad \theta(\xi,0) = 1, \quad f'(\xi,\infty) = 1, \quad \theta(\xi,\infty) = 0, \tag{10}$$

where ξ is the dimensionless streamwise coordinate, $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number, $M = \sigma B^2 x/(\rho U_\infty)$ is the magnetic parameter, $\mathrm{Re}_k = U_\infty \sqrt{k}/\nu$ is the modified local Reynolds number, $f_w = -2a/(U_\infty \nu)^{1/2}$ is the blowing (suction) parameter, and $\theta_r = (T_r - T_\infty)/(T_w - T_\infty) = -1/(\gamma(T_w - T_\infty))$ is a constant; prime denotes differentiation with respect to η.

It is worth noting that if $\gamma \to 0$ (i.e., $\mu = \mu_\infty$ is a constant), then $\theta_r \to \infty$ and Eq. (8) with $M = 0$ reduces to that given in [9]. It is also important to note that θ_r is negative for liquids and positive for gases.

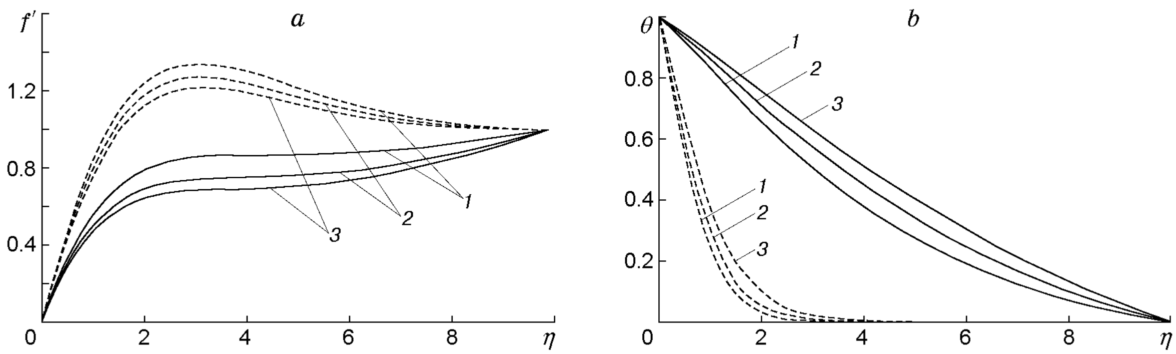

Fig. 1. Velocity (a) and temperature (b) profiles for $M = f_w = 0$, $\mathrm{Re}_k = 0.01$ and $\theta_r = -1$ (1), -0.1 (2), and -0.01 (3): solid and dashed curves refer to $\xi = 0$ and 100,000, respectively.

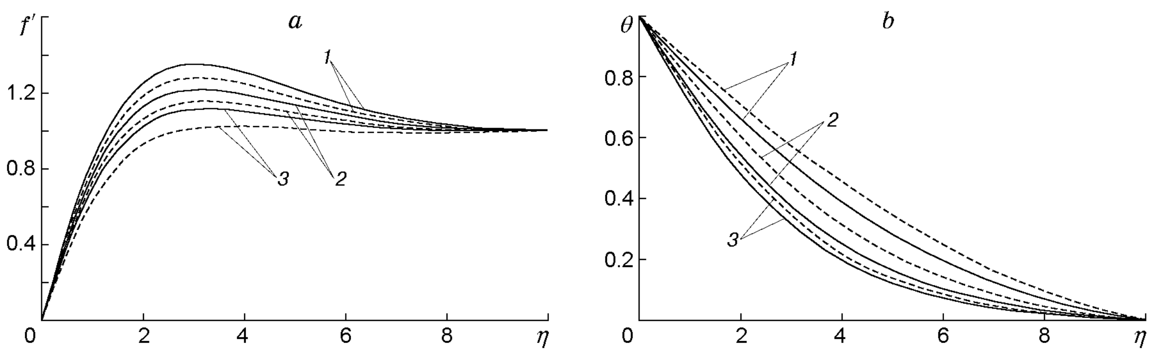

Fig. 2. Velocity (a) and temperature (b) profiles for $\theta_r = -2$, $\xi = 0.1$, $\mathrm{Re}_k = 0.01$, and $f_w = -0.3$ (1), 0 (2), and 0.3 (3): solid and dashed curves refer to $M = 0$ and 0.2, respectively.

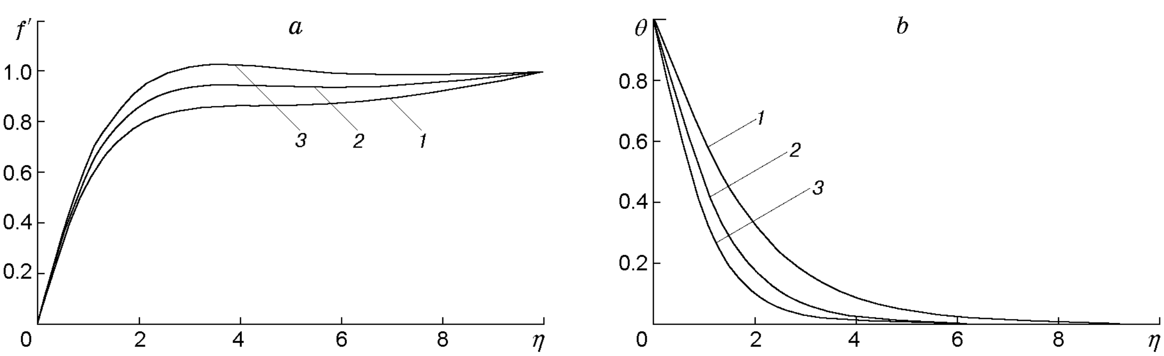

Fig. 3. Velocity (a) and temperature (b) profiles for $M = f_w = 0$, $\theta_r = -2$, $\xi = 0.1$, and $\mathrm{Re}_k = 0$ (1), 1 (2), and 2 (3).

TABLE 1

θ_r	M	f_w	$f''(\xi,0)$	$\theta'(\xi,0)$
−2.00	0	0	1.9743	−4.6371
−1.00	0	0	1.9221	−4.5062
−0.10	0	0	1.7592	−4.4241
−0.01	0	−0.3	1.7958	−4.2057
−0.01	0	−0.1	1.8443	−4.0764
−0.01	0	0	1.9125	−3.8653
−0.01	0	0.1	1.9874	−3.5892
−0.01	0	0.3	2.6651	−3.1973
−0.01	0.2	−0.3	1.8834	−4.2165
−0.01	0.2	−0.1	1.9545	−4.1078
−0.01	0.2	0	2.0213	−3.8942
−0.01	0.2	0.1	2.0924	−3.6152
−0.01	0.2	0.3	2.1815	−3.2391

The physical quantities of interest in this problem are the skin friction and the Nusselt number, which are defined by

$$C_f = 2\tau_w/(\rho U_\infty^2), \qquad \mathrm{Nu} = xq_w/(K(T_w - T_\infty)),$$

where $\tau_w = \mu_w(\partial u/\partial y)_{y=0}$ and $q_w = -K(\partial T/\partial y)_{y=0} = -KA(U_\infty/\nu)^{1/2}\theta'(\xi,0)x^{\lambda-0.5}$.

Using Eqs. (1), (3), and (8) and the dimensionless parameters indicated above, we can write

$$C_f\mathrm{Re}^{1/2} = (2\theta_r/(\theta_r - 1))f''(\xi,0), \qquad \mathrm{NuRe}^{-1/2} = -\theta'(\xi,0).$$

The governing boundary-layer equations (8) and (9) subject to boundary conditions (10) are approximated by a system of non-linear ordinary differential equations by replacing the derivatives with respect to ξ by two-point backward finite differences. These equations are integrated by shooting using the fourth-order Runge–Kutta method with a step size of 0.01. Table 1 gives numerical values of $f''(\xi,0)$ and $\theta'(\xi,0)$ for $\mathrm{Pr} = \mathrm{Re}_k = \xi = 1$ and $\lambda = 0$ and various values of the parameters θ_r, M, and f_w. It is evident that with increase in θ_r, the values of $f''(\xi,0)$ decrease and the values of $\theta'(\xi,0)$ increase. At the same time, an increase of the magnetic parameter leads to an increase in $f''(\xi,0)$ and a decrease in $\theta'(\xi,0)$. As the blowing rate increases, the values of $f''(\xi,0)$ and $\theta'(\xi,0)$ decrease, whereas an increase in suction rate leads to the opposite effect.

Results and Discussion. Calculation results for flow parameters are presented in Figs. 1–3 for $\lambda = \mathrm{Pr} = 1$ and for various values of θ_r, f_w, M, Re_k, and ξ.

Figure 1 shows velocity and temperature profiles for $\xi = 0$ (fluid flow) and 100,000 (Darcy flow). As might be expected, an increase in the parameter θ_r leads to a decrease in f' and increase in θ. The results presented here demonstrate quite clearly that θ_r, which characterizes the temperature dependence of the viscosity has a substantial effect on the drag and heat-transfer characteristics as well as on the velocity and temperature distributions in the boundary layer over a flat plate.

Curves of the horizontal velocity in the boundary layer versus the parameters M and f_w are shown in Fig. 2. It is evident that the magnetic field decreases the fluid velocity and the boundary-layer thickness. Injection increases the velocity whereas suction decreases it. An increase in magnetic-field strength leads to an increase in temperature (Fig. 2b). In addition, from Fig. 2b it follows that suction reduces the temperature while injection increases it.

The effects of Re_k on the velocity and temperature profiles are given in Fig. 3. It can be seen that with increase of Re_k, the velocity increases while the temperature decreases.

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